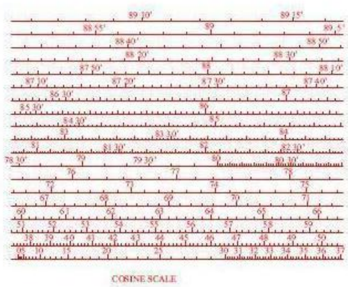
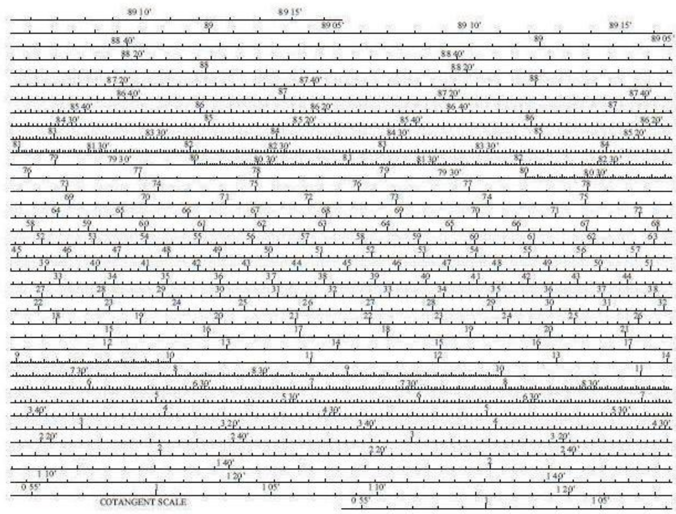
<u>Tina's Bygrave Position Line Slide Rule</u>

Once more I have been able to stand on the shoulders of giants in order to obtain my low cost, fully functional Bygrave Position Line Slide Rule. The scales were downloaded from Gary LaPook's website (https://sites.google.com/site/fredienoonan/other-flight-navigation-information/modern-bygrave-slide-rule) celebrating the exploits of Freddie Noonan and Amilia Earhart. These scales have been printed out on A3 sheets and then sellotaped next to eachother on a convenient cardboard tube. The tube needs to be big enough to accommodate the scales but not of an exact size. Results which overflow at the top and bottom of the scales have no meaning. The scales don't move once attached, all the work is done using the cursors. A large celluloid sheet wide enough to encompass the Cosine scale and long enough to wrap around the cardboard tube is used for the major cursor. A crosshair is drawn or attached to this near to the left hand side as shown above. A second thin strip of celluloid with a crosshair is wrapped around the major cursor to produce the minor cursor. The major cursor is free to move horizontally and around the tube. The minor cursor can move similarly and independently of the major.

In order to cover the cosine scale from 0 degrees (Cosine =1) to 89 25' 37" (Cosine =0.01) two decades of logarithms are required. In order to cover the cotangent scale from 0 34' 23" (Cotangent = 100) to 89 25' 37" (Cotangent = 0.01) four decades of logarithms are required. Gary LaPook obviously agrees because that is what he has provided, although it should be noted that the uppermost line of the Cosine scale and the upper and lowermost lines of the Cotangent scale have been omitted, probably because they are not of use. Gary also placed two cotangent scales side by side so that there would not be a problem with results occurring off the scale.

The user needs to be aware that the cotangent scale has no origin. As the angle tends to 0 degrees the cotangent tends to infinity and as the angle tends to 90 degrees the cotangent tends to zero but never reaches it. Attempting to multiply two cotangents together will be complicated as a result. Fortunately the equations used in celestial navigation do not require this. Should a user want to multiply or divide cotangents it is possible by using 45 degrees as the origin since the cotangent of 45 degrees is 1. The cosine scale does have an origin at 0 degrees with a cosine of 1. This can be safely used when setting cosines into the slide rule.





Bygrave's Position Line Slide Rule was designed to solve equations used by Aviators and Mariners for celestial navigation. It is very well described in a paper by Ronald van Riet available at http://www.rechenschieber.org/PositionLineSlideRules.pdf. Gary LaPook, Ronald van Riet and of course Captain L.G. Bygrave are the giants I referred to earlier.

Ronald's description of the use of the Bygrave boils down to the solution of four simple equations.

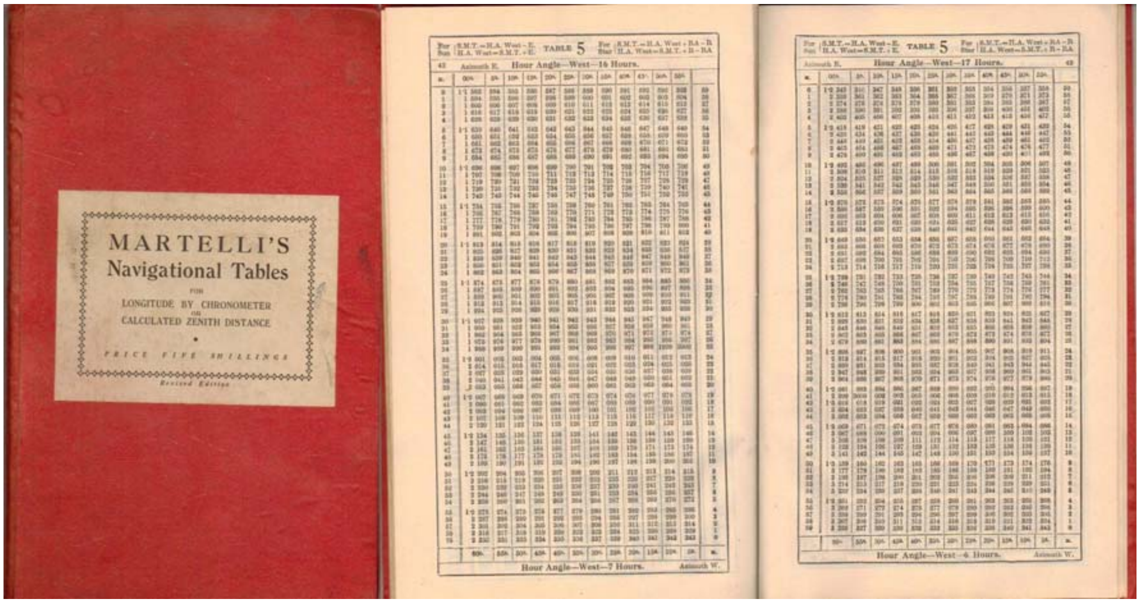
Equation 1

Equation 1:- 1/cot(d)= cos(LHA)/cot(X)

Which for my purposes translates to equation 5 :- cot(X)=cos(LHA)*cot (d)

Where d = the declination of the celestial object (star) being used for navigation purposes. This is its height above the Equatorial Plane (also known as the Equinoctial Plane) measured in degrees. This is available from tables.

LHA = GHA (of the celestial object) – The Longitude of the navigator's assumed position (West being positive, East being negative). Where GHA = the Greenwich Hour Angle which is the position past the plane of the Greenwich Meridian in degrees. This, as shown in the image below can be found in tables such as Martelli's Navigational Tables.



The slide rule finds cosines for angles between 0 and 89 16'. If the value of LHA is outside this range then the following rules must be used to find the correct angle within the range.

If 90 < LHA <= 180 then LHA(new) = 180 - LHA and the cosine will be negative.

If 180 < LHA <= 270 then LHA(new) = LHA - 180 and the cosine will be negative.

If 270 < LHA <= 360 then LHA(new) = 360 – LHA and the cosine will be positive.

X is the Intermediate Angle and the required result of equations 1 and 5.

Using the homemade Bygrave slide rule the Intermediate Angle, X is obtained by setting the major cursor at the origin (0 degrees since cosine of 0 is 1) of the cosine scale and the minor cursor at Local Hour Angle (LHA). With this proportion set the major cursor is then moved to the Declination (d) on the cotangent scale and the value X is then read under the minor cursor.

Equation 2

Equation 2:- Y=90 - Latitude of navigators assumed position + X

If 0 < Y < 90 (Ignoring the sign of Y) then Y does not need to be changed.

If 90 < Y < 180 (Ignoring the sign of Y) then Y(new) = 180-Y

Y is the second intermediate angle and the required result of equation 2.

The slide rule won't help you with addition and subtraction, use a pen.

Equation 3

Equation 3 :- cos(X)/cot(LHA) = cos(Y)/cot(Az)

Which for my purposes translates to equation 6 :- cot(Az) = cos(Y)*cot(LHA)/cos(X)

Where Az = Azimuth the angle between the nearest pole (North or South) and the celestial body. This is the required result of equations 3 and 6.

The method of solving this equation depends on the values of Y and X and the physical position of their cosines on the cosine scale.

<u>When Y is Greater than X</u>

When Y is greater than X the result of the divison of cos (Y) by cos (X) is found and multiplied by cot(LHA).

Using the homemade Bygrave slide rule set the major cursor to X and the minor cursor to Y on the cosine scale, move the major cursor to LHA on the cotangent scale and read the resulting Azimuth (Az) under the minor cursor. If moving the minor cursor to Y places it off the left side of the major cursor then reverse the setting. Place the major cursor to Y and the minor cursor to X on the cosine scale, move the major cursor so that the minor cursor is at LHA on the cotangent scale and read the resulting Azimuth (Az) under the major cursor.

<u>When X is Greater than Y</u>

When X is greater than Y result of the divison of cos (X) by cos (Y) is found and cot (LHA) is divided by this value.

Using the homemade Bygrave slide rule set the major cursor to X and the minor cursor to Y on the cosine scale, move the major cursor so that the minor cursor is at LHA on the cotangent scale and read the resulting Azimuth (Az) under the major cursor. If moving the minor cursor to Y places it off the left side of the major cursor then reverse the setting. Place the major cursor to Y and the minor cursor to X on the cosine scale, move the major cursor to LHA on the cotangent scale and read the resulting Azimuth (Az) under the minor cursor.

<u>Equation 4</u>

Equation 4:- cos(Az)/ cot(Y) = 1/cot(a)

Which for my purposes translates to equation 7:- cot(a)=cot(Y)/cos(Az)

Where a is the Altitude and the required result of equations 4 and 7.

Using the homemade Bygrave slide rule set the major cursor to the origin of the cosine scale and the minor cursor to Az. Move the major cursor so that the minor cursor is at Y on the cotangent scale and read the Altitude (a) under the minor cursor.

If all you want to do is complete the required calculations using a position line slide rule there is no need to read on. If you want to understand what the results are for then the rest of this article may give you a clue.

What is the altitude needed for?

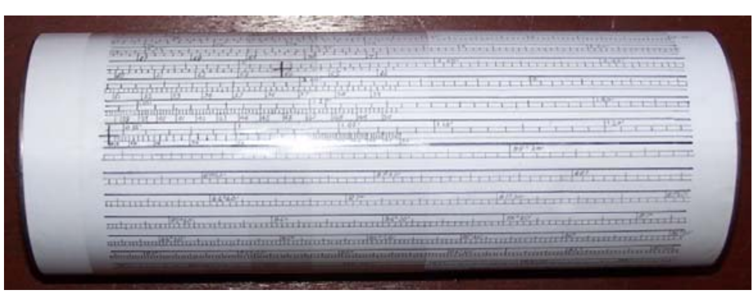
The Altitude of the celestial body is the angle between the celestial body and the navigator's horizontal. The navigator will use a sextant to find the altitude of a known celestial body for which the necessary tables are available in his nautical almanac. Once the expected altitude of the selected star for the navigator's assumed location is known (from the above equations) a circle can be drawn on a chart showing all the possible positions of the navigator (this is the position line) for which this altitude is possible. The difference between this calculated altitude and the actual measured altitude in minutes of arc is the distance of the navigator (North or South) from this position line measured in Nautical Miles. By finding the altitude of a second star a second circle can be drawn which will intersect the first at two points. One of the points found should be obviously wrong but to remove doubt a third star's altitude could be found to settle the issue. The navigator now knows how far he is from his assumed location. If navigating during the day only one celestial object (the sun) may be visible therefore the second and third altitudes will not be available to narrow down the navigator's position.

Latitude and Longitude.

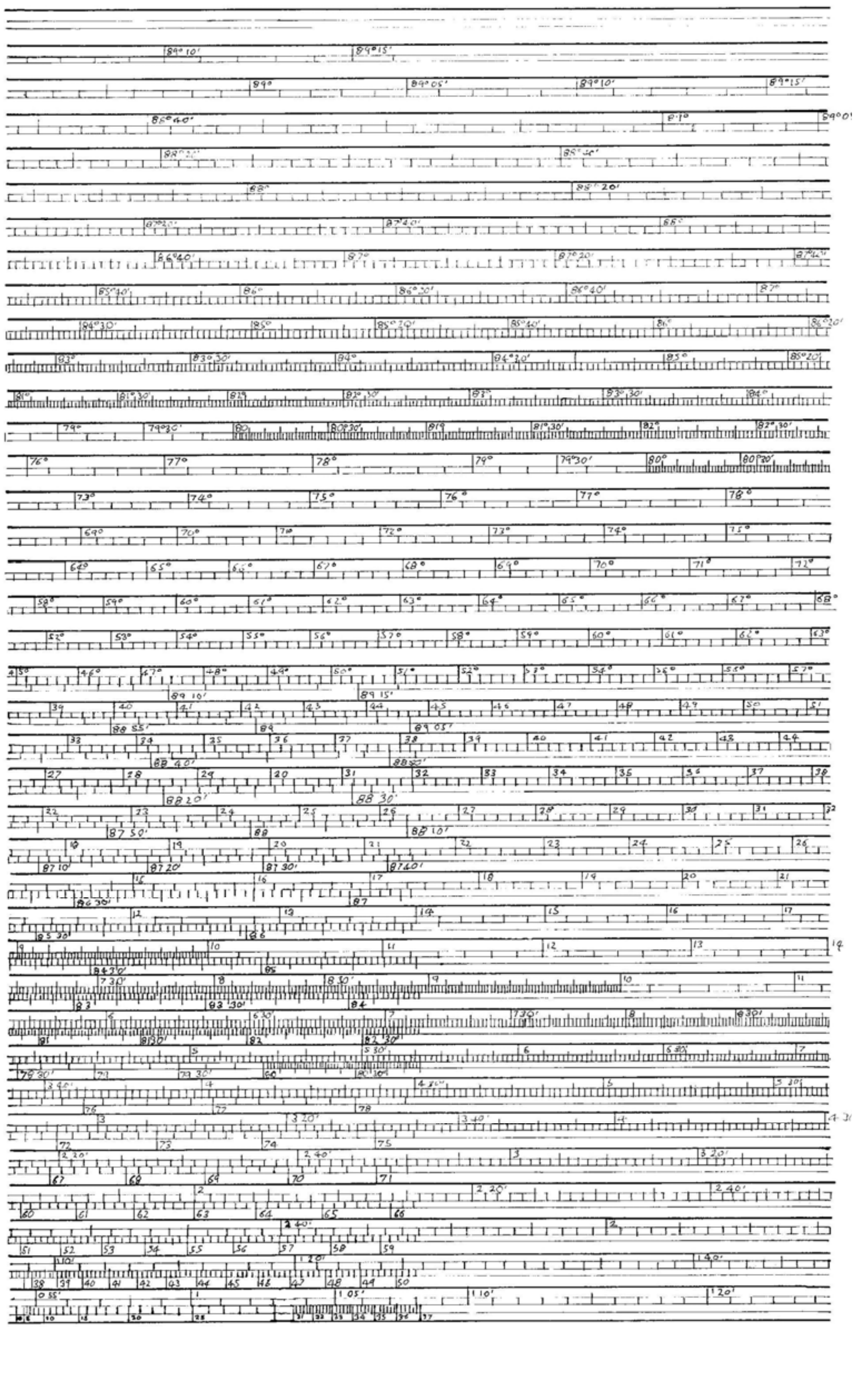
When referring to Latitude and Longitude in this article I am not referring to the actual latitude and longitude of the navigator. If the navigator knew this he wouldn't need to do any calculations. What the navigator does is make an educated guess of his Latitude and Longitude (dead reckoning) and feeds these values into the equations above. By making two or three guesses and plotting the resulting positions on a chart he will find that they lie on a line (actually the circumference of a circle of large radius). This was first discovered by Captain Thomas H Sumner in 1837 and is still sometimes referred to as the Sumner Line but more familiar to slide rule collectors as the Position Line referred to in the title. By using a parallel ruler on the chart the position line can be moved to intersect the navigator's intended destination. The navigator can then correct his course to follow the new position line.

Mark II

The natural extension of this design of Position Line Slide Rule is to combine the Cosine and Cotangent scales onto a single scale sheet. This was my next development.



The second prototype is shown above. The scale I have drawn is shown below. The scale consists of two identical scales side by side drawn in the style of a Gridiron. Each scale consists of four decades of logarithms covering the cotangents from 0 degrees 55 minutes and 89 degrees 15 minutes. A cosine scale produced with two decades of logarithms occupies the bottom left quadrant.



Position Line Slide Rule Conclusions.

I am not a navigator therefore it is quite possible that my understanding and therefore my explanation of a navigator's reasoning may be flawed. It is only the slide rule design and its mode of use which I am prepared to guarantee.

The design as shown above is very simple to make, easy to use and yet still accurate. The main flaw in the design of the original Bygrave is stated as being the possibility that the two cylinders carrying the scales might move accidentally with respect to each other during a calculation and cause a dangerous error in the result. In this design the minor cursor remains in place with respect to the major cursor when the major cursor is moved therefore the aforementioned problem does not occur.

The scale I drew for the second version of the Position Line Slide Rule is definitely inaccurate. Please don't use it on any round the world yachting cruises. This scale has been made solely to demonstrate the feasibility of this design, a job I believe it does successfully.

At this point I would like to observe that there is no reason why a slide rule of this design should be cylindrical. A tube with an elliptical or any other smoothly curving cross section would work just as well.

Now that I have my Position Line Slide Rule all I need is a sextant and I will be able to find out if my house has drifted off course.